On the Downlink SINR and Outage Probability of Stochastic Geometry Based LTE Cellular Networks with Multi-Class Services

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Abstract—Due to the increasing number of irregularly spaced base stations (BSs) as well as the intrinsic random channel environment, modeling and analysis of cellular networks using classical hexagonal cell shapes is becoming ever more impractical. Therefore, stochastic geometry models having the ability to picture near realistic situations is gaining a wide acceptability for evaluating cellular network performance. In light of this, this paper presents a simulation-based investigation on the downlink signal-to-interference-noise-ratio (SINR) and the outage probability of orthogonal frequency division multiple access (OFDMA)-based long term evolution (LTE) cellular systems using stochastic geometry with multi-class services. Locations of BSs are modeled using both Poisson point process (PPP) as well as hard-core Poisson process (HCPP). Moreover, a computationally efficient method is proposed for capturing the effect of inter-cell interference in such stochastic geometry based cellular networks. Network performance including the outage probability of various multi-class services under varying shadow fading scenario and BS density is evaluated using Monte Carlo simulations, and compared with that of the traditional hexagonal models. Simulation results clearly demonstrates the over optimistic network performance of hexagonal model, while the most realistic HCPP model provides a compromise between the hexagonal and the PPP models.

Index Terms—Stochastic Geometry, LTE, Poisson Point Process, Hard-core Poisson Process, outage probability.

I. INTRODUCTION

To meet the ever increasing demand of subscribers for diverse applications, cellular network operators are deploying more and more base stations (BSs) near the traffic hot spots. Consequently, the cell deployment layout of modern cellular networks is increasingly becoming random and thus moving further away from the regular hexagonal pattern. As a matter of fact, the simplified Wyner model [1] is not close enough to analyze such complicated modern cellular systems. The two dimensional (2D) regular hexagonal model assuming regularly spaced hexagonal cells as shown in Fig. 1(a) also provides too optimistic results and does not reflect the inherent random phenomena prevailing in real cellular networks. Thus a query for finding an effective alternative to these models followed to the proposition of stochastic geometry model [2]. Stochastic geometry model allows to integrate the random shapes of cells

by modeling the distribution of nodes over a terrain according to some particular random distribution. The large amount of uncertainty in situations that can occur in a network thus can be analyzed using a probabilistic approach. The spatial distributions of the aforementioned nodes which are eventually the BSs can be modeled using Poisson point process (PPP) [3]. On the other hand, recent studies identified that for many existing networks, hard-core poisson process (HCPP) based stochastic modeling could capture a more realistic spatial distribution of BSs leading to improved accuracy in performance evaluation [4].

The distribution of downlink signal-to-interference-noise-ratio (SINR) for an orthogonal frequency division multiple access (OFDMA) based cellular network using hexagonal model was evaluated in [5], while completely ignored the irregular placement of BSs. On the other hand, the downlink SINR for a heterogeneous network using only PPP based stochastic model was investigated in [6]. Authors in [7] formulated a tractable methodology in analyzing the coverage probability and the achievable data rates. Whereas an optimal SINR-based coverage enhancement scheme employing increased BS density was analyzed in [8]. Another analytical modeling based downlink outage analysis for cognitive heterogeneous cellular networks considering PPP distributed BSs was presented in [9]. However, none of these papers considered neither multi-class service scenario nor the more realistic HCPP model. Avoiding the adoption of HCPP distributed geometry is mainly attributed to the extreme complexity in developing analytical models.

In this paper, a simulation based investigation of the downlink performance of OFDMA-based long term evolution (LTE) cellular networks using stochastic geometry models with multi-class services is presented. Both PPP and HCPP stochastic models are investigated under the considered network scenario. Furthermore, we propose a computationally efficient technique for calculating the inter-cell interference and define an effective interfering zone for stochastic geometry based models. Network performance for both PPP and HCPP models is evaluated in terms of received SINR and the outage probability, and compared to that of traditional

(a) Regularly spaced hexagonal BSs   (b) BSs are modeled as PPP   (c) BSs are modeled as HCPP
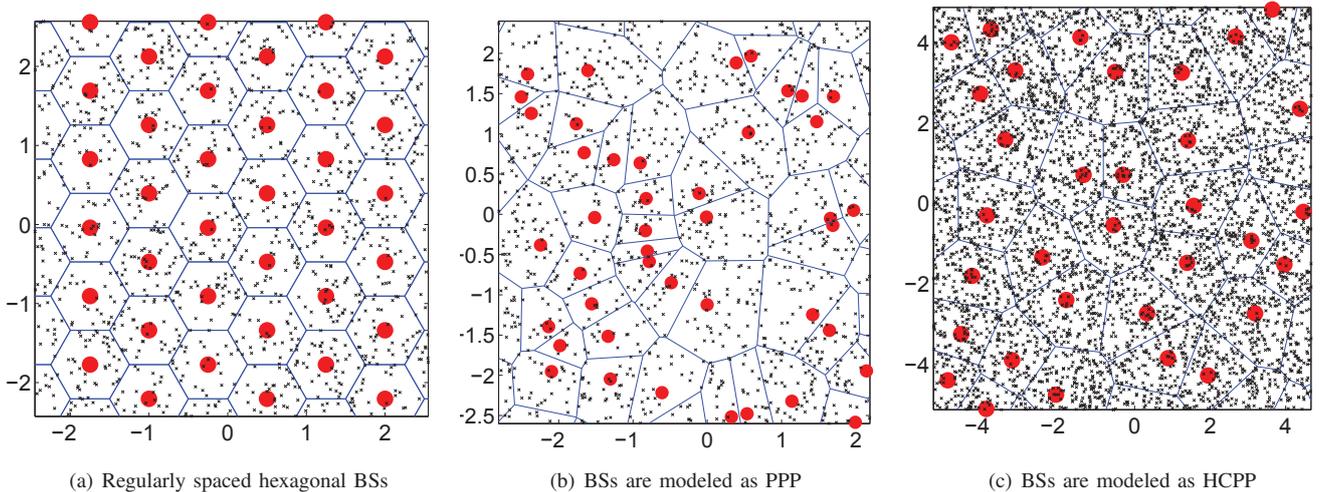
Fig. 1: Various models for cellular networks. The big circles are the location of BSs. The scale in both axes is in km.

regularly spaced hexagonal cell model. Impact of various data rates, intensity of shadow fading and the BS density is also presented.

The rest of this paper is organized as follows: Section II introduces the system model with the key features. Simulation results and an in depth analysis is presented in section III, whereas the key findings are summed up at section IV.

II. SYSTEM MODEL

We consider the downlink of an OFDMA based LTE cellular network serving N different service classes denoted by $C_1, C_2, ..., C_N$ classes of services. In this paper, it is assumed that the service classes differ only in the required data rates. BSs are deployed using omnidirectional antennas and universal frequency reuse is considered.

A. Network Layout

For accounting the irregularly spaced BSs, this paper models the locations of BSs using both PPP and HCPP based stochastic geometry models. If the spatial distribution of BSs over the terrain is of PPP with density λ_{BS}, then the number of points $\Phi(A)$ in a bounded set $A \subset \mathbb{R}^2$ has Poisson distribution with mean $\lambda_{BS}|A|$ and is given by

$$\mathbb{P}(\Phi(A) = n) = e^{-\lambda_{BS}|A|} \frac{(\lambda_{BS}|A|)^n}{n!} \qquad (1)$$

Although this model closely resembles the real-life random BS locations and coverage areas, it does not put any kind of restriction on the minimum distance between two BSs, i.e., two BSs can be very close to one another as seen in Fig. 1(b). However, in practice, BSs must have some geographic distance from neighboring BSs. For emulating such practical phenomena, we then apply a thinning process on the PPP model such that no two BSs can stay closer than a certain distance h resulting in a HCPP distributed BSs. A snapshot of HCPP based network model obtained by applying a thinning process in the network of Fig. 1(b) is shown in Fig. 1(c).

In particular, we model the locations of BSs using a Matérn HCPP resulting a modified BS density $\hat{\lambda}_{BS}$ as given by [10]

$$\hat{\lambda}_{BS} = \frac{1 - \exp(-\lambda_{BS}\pi h^2)}{\pi h^2} \qquad (2)$$

Locations of UEs are also modeled by an independent PPP with parameter λ_{UE}. On the other hand, UEs are assumed to be associated with their corresponding geographically closest BSs.

B. Channel Model

This paper adopts the WINNER+ non line of sight (NLOS) urban macro-cell path-loss model [11]. Thus the path-loss P_L at a distance d in dB is given by

$$P_L = (44.9 - 6.55 \log_{10} h_{BS}) \log(d) \\ + 5.83 \log_{10} h_{BS} + 14.78 + 34.97 \log_{10} f_c \qquad (3)$$

where f_c is the carrier frequency and h_{BS} is the height of the BS. For $h_{BS} = 25m$ and $f_c = 2GHz$, the equation simplifies to

$$P_L = 138.4 + 35.74 \log_{10}(d) \qquad (4)$$

Shadow fading is also considered and modeled as lognormally distributed random variable with zero mean and standard deviation σ dB. Due to the inherent robustness of OFDM systems, small scale fading is ignored.

C. Resource Block Allocation

Received SINR at u^{th} UE U_u of jth class located in i^{th} BS can be given by

$$\gamma_{i,u}^{j} = \frac{P_{i,u}^{j}}{\mathcal{I}_{i,u}^{j,inter} + \mathcal{I}_{i,u}^{j,intra} + \mathcal{P}_N} \qquad (5)$$

where $P_{i,u}^{j}$, $\mathcal{I}_{i,u}^{j,inter}$, $\mathcal{I}_{i,u}^{j,intra}$ and $\mathcal{P}_N$ are the received power, inter-cell interference, intra-cell interference and the additive white Gaussian noise (AWGN) power respectively. Due to

the orthogonality of LTE RBs, intra-cell interference becomes zero. Now, considering adaptive modulation and coding (AMC), received SINR $\gamma_{i,u}^j$ can then be mapped to the spectral efficiency (SE) given in bps/Hz as below [12]

$$\psi_{i,u}^j = \begin{cases} 0 & \text{if } \gamma_{i,u}^j < \gamma_{min} \\ \xi \log_2(1 + \gamma_{i,u}) & \text{if } \gamma_{min} \leq \gamma_{i,u}^j < \gamma_{max} \\ \psi_{max} & \text{if } \gamma_{i,u}^j \geq \gamma_{max} \end{cases} \quad (6)$$

where $0 \leq \xi \leq 1$, γ_{min}, ψ_{max} and γ_{max} are the attenuation factor accounting implementation loss [12], minimum SINR, maximum SE and the SINR at which ψ_{max} is achieved. Then the number of required RBs for the UE for maintaining a particular data rate can be estimated by

$$\beta_{i,u}^j = \left\lceil \frac{R_{i,u}^j}{W_{RB} \psi_{i,u}^j} \right\rceil \quad (7)$$

where $R_{i,u}^j$ is the required data rate in bps, W_{RB} is the bandwidth per RB in Hz (e.g., 180 kHz in LTE), and $\lceil x \rceil$ is the nearest integer equal to or larger than x. On the other hand, if the number of RBs per UE is set fixed, (6) - (7) can then be used for estimating the required SINR for a given data rate.

D. Outage Probability

Let $\gamma_{j,th}$ is the required minimum (i.e., threshold) SINR for a UE from class j for its effective communication. Then the outage probability of uth UE U_u from j^{th} class C_j located in i^{th} BS can be given by

$$P_{i,u}^{j,out} = Pr\{\gamma_{i,u}^j \leq \gamma_{j,th} | U_u \in C_j\} \quad (8)$$

Outage probability of UE U_u from any class in BS i can then be written as

$$P_{i,u}^{out} = \sum_{i=1}^{N} Pr\{\gamma_{i,u}^j \leq \gamma_{j,th} | U_u \in C_j\} Pr\{U_u \in C_j\} \quad (9)$$

where $Pr\{U_u \in C_j\}$ is the probability that user u is from class C_j. The overall outage probability of the entire network can now be evaluated by taking the network-wide average over all UEs and all BSs.

III. SIMULATIONS, RESULTS AND DISCUSSIONS

A. Simulation Setup

We consider a network covering a geographical area of 20x20 km². Monte Carlo simulation is then used for investigating the network performance. BSs are equipped with omnidirectional antennas with carrier frequency = 2GHz, channel bandwidth per BS = 5MHz (i.e., 25 RBs) and BS transmit power = 20W. This gives a transmit power per RB equal to 0.8W. AMC code set parameters $\xi = 0.75$, γ_{min} = -6.5dB, γ_{max} = 19dB and ψ_{max} = 4.8bps/Hz are chosen in reference to the 3GPP LTE suggestions [12]. On the other hand, thermal noise power in dB is calculated using $\mathcal{P}_N = -174 + 10\log_{10}(\Delta f)$, where (Δf) is the channel bandwidth in Hz. Network performance is evaluated
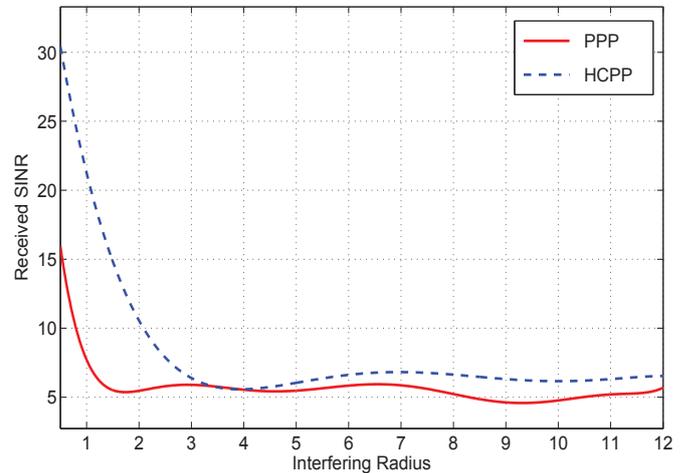

Fig. 2: Estimation of effective interfering zone for stochastic models.

under worst case scenario considering that all the RBs in all BSs are in use, which corresponds to the severe most inter-cell interference scenario. Thus the presented SINR and the outage are essentially the lower bound and the upper bound respectively.

For creating a multi-class scenario, three classes of real-time constant bit rate services having data rates equal to 64 kbps, 384 kbps and 512 kbps including packet headers and payloads are considered. Without losing the generality, the proportions of users from these classes are considered equal to 50%, 30% and 20% respectively, which are uniformly distributed over the entire network. It is also assumed that only one RB can be allocated to a UE from any class. Thus, using (6)-(7), we calculate the threshold SINR for the three classes equal to -4.1 dB, 7.9 dB and 11.1 dB respectively, i.e., any UE receiving SINR below its threshold SINR will face outage. Unless otherwise specified, BS density $\lambda_{BS} = 2$, HCPP thinning radius $h = 0.5$ km and shadow fading standard deviation $\sigma = 8$ dB are assumed for the simulations.

B. Result Analysis

1) Estimation of Interfering Zone: It is widely accepted that inclusion of inter-cell interference from two neighboring tiers is sufficient for calculating SINR in a regularly spaced hexagonal deployment scenario. However, there is no such thumb rule in case of stochastic geometry based network models and the most appropriate procedure is to include the interference from all BSs in a network, which is overwhelmingly computation intensive. In this paper, we proposed an alternative efficient, yet effective technique for considering inter-cell interference. In Fig. 2, we present the received average SINR in UEs with the interfering radius. The interfering radius defines the region in which the located BSs create interference for the UEs in the reference cell. From the figure, it is evident that after certain distance, SINR becomes nearly constant for both the PPP and HCPP models, which indicates that exclusion of interference from BSs outside of this radius will not affect the performance evaluation. We call this radius as the interfering
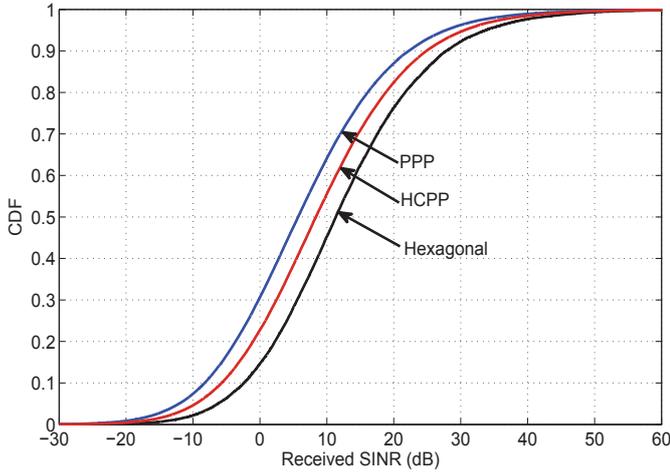
Fig. 3: CDF of received SINR for hexagonal, PPP and HCPP based cellular networks.
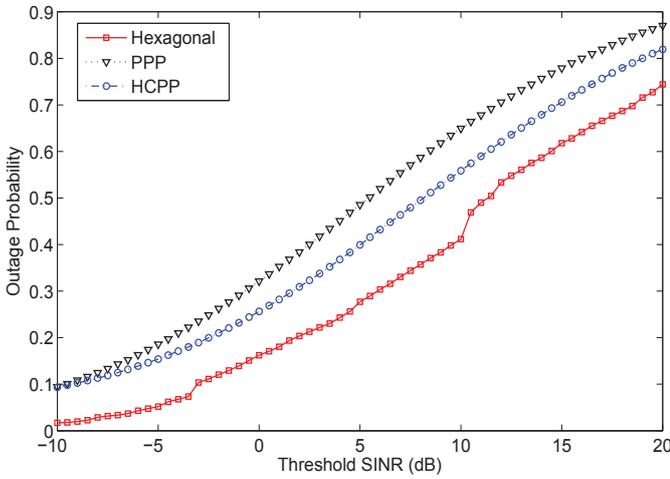

Fig. 5: Outage probability with shadow fading standard deviation σ under multi-class services in HCPP based model.

Fig. 4: Comparison of outage performance among hexagonal, PPP and HCPP based cellular networks.

Fig. 6: Outage probability with BS density λ_{BS} under multi-class services in HCPP based model.

radius defining an effective interfering zone, which is found to be approximately equal to 2.76 km and 4 km for PPP and HCPP models respectively. Exclusion of BSs outside of interfering zone significantly reduces the computational load. It is to be noted that the interfering zone is significantly lower for PPP model compared to that of HCPP model. This is because of the closer proximity of BSs in PPP distribution than those in HCPP model. The rest of the results presented in this paper are evaluated considering the interfering zone evaluated from Fig. 2.

2) Comparison among the Hexagonal and Stochastic Models: Fig. 3 presents the cumulative distribution function (CDF) of received SINR demonstrating a glance of the distribution of SINR throughout the considered network models. For fair comparison, number of BSs in the hexagonal model is kept equal to that of the stochastic models. After the regularly spaced placement of these BSs in a hexagonal pattern gives an approximate cell radius equal to 0.5 km. From the figure,
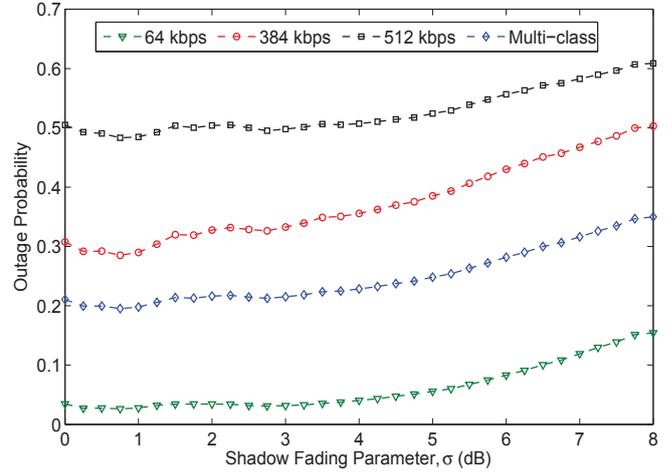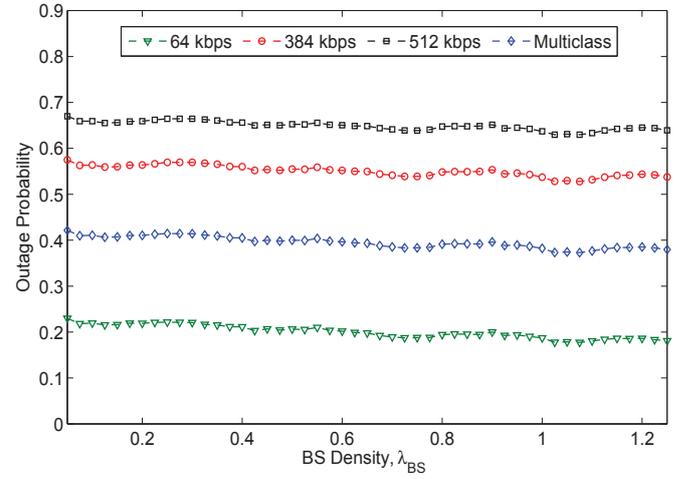
a clear distinction in SINR distribution is observed between the deterministic hexagonal model and the stochastic geometry models (PPP and HCPP). Hexagon model keeps its optimistic nature distributing comparatively stronger SINR among UEs which ranges from -33.3 dB to 60 dB. On the other hand stochastic models cover a larger range of SINR in their distribution that covers from -91.3 dB to 87.7 dB. The slightly tapered region of the HCPP and PPP curves refers to a range of SINR (-40 dB to -20 dB) implies fewer UEs than the other regions of SINR.

On the other hand, Fig. 4 illustrates the outage probability of the three network models with the varying threshold SINR. As expected, outage is found to increase with the increase of required threshold SINR. This figure also once again demonstrates the optimistic nature of hexagonal cell model in which UEs suffer from much lower outage, while the most pessimistic performance is obtained by PPP model. Although PPP and HCPP are both stochastic models, HCPP results

lower outage than that of PPP. The closely spaced BSs in PPP compared to HCPP give results to weaker SINR for UEs in PPP leading to higher outage.

3) Multi-Class Analysis: Fig. 5 illustrates the outage probability of an HCPP based cellular network with multi-class services. The figure depicts the outage of the individual service classes as well as the overall outage of the network with the varying shadow fading standard deviation σ. Varying σ corresponds to various types of terrain and serving regions, such as urban, suburban and rural regions. In general, with the increase of σ, the randomness in the received SINR level increases resulting in a higher outage probability, which is also evident from the figure. We also observe that the trend of outage with respect to σ does not depend much on data rates. But it shifts upward with the increasing data rates because of the higher SINR requirement at higher data rates.

In Fig. 6, the impact of BS density λ_{BS} on the outage probability under multi-class scenario of HCPP based model is demonstrated. The λ_{BS} was varied up to $\frac{1}{\pi h^2}$. This is the maximum density of BSs for this network model. An attempt of deploying more BSs in per unit area will be removed by the thinning process. From the figure, it is observed that the increasing λ_{BS} does not have any effective influence over outage. We explain this using the definition of SINR. The effect of increasing the density of of BSs can affect SINR simultaneously in two ways. Firstly, it increases received power by a UE from its serving BS. Secondly, the denser BSs also result to stronger interfering signal. These two simultaneous effects compensate each other leaving a little influence on the outage probability.

IV. Conclusions

This paper has investigated the downlink performance of stochastic geometry based OFDMA based LTE cellular networks with multi-class services. The spatial randomness of cellular networks has been modeled using both PPP and HCPP stochastic geometry models, and the performance is also compared with that of traditional hexagonal cell model. Furthermore, a computationally efficient method for defining interfering zone is proposed and investigated for stochastic geometry models. Performance of various network models under multi-class scenario is evaluated in terms of received SINR and the outage probability. Outage probability is found to have increasing trend with higher data rates and shadow fading intensity, while remains approximately unaffected with the changing BS density. It has also been identified that the PPP and the hexagonal models present the most pessimistic and the most optimistic network performance respectively, while the performance of the more realistic HCPP model lies in between them.

Our future works will focus on the analytical modeling and the verification of the network performance with the simulation results presented in this paper.

References

[1] A. D. Wyner, "Shannon-theoretic Approach to a Gaussian Cellular Multiple-Access Channel," *IEEE Transactions on Information Theory*, vol. 40, no. 6, pp. 1713-1727, Nov 1994.

[2] F. Baccelli, "Stochastic Geometry and Architecture of Communication Network," *Telecommunication System*, vol. 7, no. 1-3, 1997, pp. 209-227.

[3] A. Guo and M. Haenggi, "Spatial Stochastic Models and Metrics for the Structure of Base Stations in Cellular Networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 11, Nov 2013.

[4] A. Tukmanov, Z. Ding, S. Boussakta and A. Jamalipour, "On the Impact of Network Geometric Models on Multicell Cooperative Communication Systems," *IEEE Wireless Communications*, vol. 20, no. 1, pp. 75-81, Feb 2013.

[5] C. Seol and K. Cheun, "A Statistical Inter-Cell Interference Model for Downlink Cellular OFDMA Networks Under Log-Normal Shadowing and Multipath Rayleigh Fading," *IEEE Transactions on Communications*, vol. 57, No. 10, Oct 2009.

[6] S. Mukherjee, "Distribution of Downlink SINR in Heterogeneous Cellular Networks," *IEEE Journal on Selected Areas in Communications,* vol. 30, No. 3, Apr 2012.

[7] J. G. Andrews, F. Baccelli, R. K. Ganti, "A Tractable Approach to Coverage and Rate in Cellular Networks," *IEEE Transactions on Communications*, vol. 59, No. 11, Nov 2011.

[8] T. Samarasinghe, H. Inaltekin and J. S. Evans, "Optimal SINR-based Coverage in Poisson Cellular Networks with Power Density Constraints," *Proc. of IEEE Vehicular Technology Conference (VTC)*, pp. 1-5, Sep 2013.

[9] F. H. Panahi and T. Ohtsuki, "Stochastic Geometry Based Analytical Modeling of Cognitive Heterogeneous Cellular Networks," *IEEE International Conference on Communications (ICC)*, pp. 5281-5286, 10-14 June 2014.

[10] D. Stoyan, W. S. Kendall, and J. Mecke, "Stochastic Geometry and its Applications." *Wiley, Chichester, 2nd edition, 1995.*

[11] "D5.3: WINNER+ Final Channel Models," *Wireless World Initiative New Radio WINNER+, June 2010.*

[12] 3GPP TR 36.942 Ver. 11.0.0 Rel. 11, "Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Radio Frequency (RF) system scenarios," *Technical Report*, Sep 2012.