Downlink Analysis of Cellular Networks with Distributed Antenna Systems for Non-Uniform User Distribution Using Stochastic Geometry

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Abstract—Distributed antenna system (DAS), consisting of multiple antenna elements (AEs) serving a single base station (BS), is a promising scheme for long term evolution (LTE) cellular systems. In various studies, DAS based networks with uniformly distributed users have been shown to yield higher capacity and reliability. However, to the best of our knowledge, non-uniform user distribution has not been taken into account in previous research on DAS. In light of this, this paper proposes and investigates various network layouts incorporating DASs, including uniform and non-uniformly distributed AEs and BSs. Both homogeneous and inhomogeneous Poisson Point Process (PPP) are utilized for modeling the locations of users, AEs and BSs. Performance of the proposed network layouts are evaluated in terms of downlink signal-to-interference-plus-noise-ratio (SINR) and outage probability using extensive Monte Carlo simulations. Comparison of these metrics with those of the conventional uniformly distributed AEs demonstrates the superiority of the proposed non-uniform deployment of network elements.

Index Terms—distributed antenna system; Poisson Point Process; non-uniform user distribution; stochastic geometry.

I. INTRODUCTION

Demand for higher capacity and throughput is putting the present network convention, which consists of a centralized antenna or base station (BS) catering to a particular area of cellular coverage, under increased pressure. Furthermore, in a centralized BS system, cell edge users suffer from lower signal strength as well as higher inter cell interference from other cells. Such a situation necessitates the use of distributed antenna system (DAS) with spatially distributed antenna elements (AEs). DAS provides a promising solution to these setbacks, increasing received power in cell edges, thereby improving the downlink signal-to-interference-and-noise-ratio (SINR) and consequently capacity and outage probability [1] - [4]. One of the major advantages of DAS is that it decreases the statistical distance of a user element (UE) from its serving AE and thereby improves the received signal quality significantly [5].

Authors in [6] - [8] analyzed the DAS outage performance and capacity assuming deterministic equally spaced AEs. However these models did not consider random shapes of cells as well as random distribution of AEs over a terrain. DAS performance was analyzed with random spatial distribution of AEs using stochastic geometry in [5], [9]. On the other hand, the authors in [9] formulated downlink outage probability as well as system capacity considering random location of AEs. Downlink outage performance with respect to shadow fading channels was analyzed in [8], while the authors in [10] analyzed the uplink outage probability.

All these previous works on DAS considered uniformly distributed user elements (UEs). However, in practical scenario, users are not distributed uniformly over the terrain [11] - [12]. In the area covered by a network, there are certain places where the density of users is notably greater, i.e. commercial areas, markets, stadiums etc. and afterwards it slowly tails off towards suburban and rural areas. This paper investigates the downlink performance of orthogonal frequency division multiple access (OFDMA)-based long term evolution (LTE) cellular networks with DAS considering non-uniformly distributed UEs over the network area. Thus this paper proposes and investigates different access point layouts considering uniform and non-uniform distributions of AEs and BSs in various combinations. Moreover, performance of the proposed network layout with DAS for non-uniform UEs is compared with that of conventional cellular systems with no DAS. The downlink performance for all the models is evaluated in terms of received SINR and outage probability. In addition, impact of threshold SINR, shadow fading, AE density and BS density on outage probability for all the network models are also analyzed. We use stochastic geometry methods viz. homogenous and inhomogeneous Poisson Point Process (PPP) for modeling uniform and non-uniform network nodes respectively.

The rest of this paper is organized as follows: Section II introduces the system model with the key features. Simulation results and an in-depth analysis is presented in section III, whereas section IV contains a summary of the key findings.

II. SYSTEM MODEL

In this paper, we consider the downlink of an OFDMA-based LTE network. BSs and AEs of DAS are deployed considering omnidirectional antennas and universal frequency reuse is assumed. In our study, we assume that channel state information (CSI) is not available at the transmitter and each

(a) Uniform BSs with no DAS   (b) Non-uniform BSs with no DAS   (c) Uniform BSs with uniform DAS   (d) Non-uniform BSs with non-uniform DAS
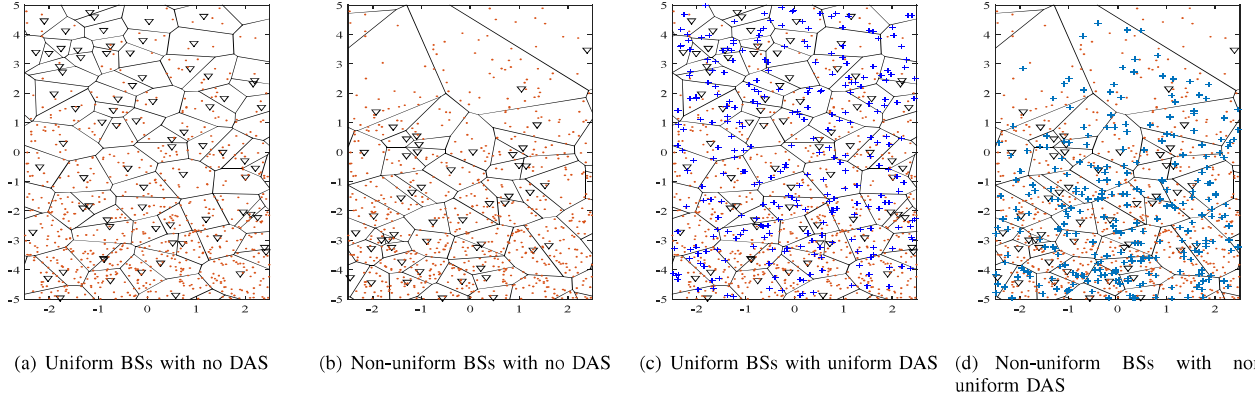
Fig. 1: Various network layouts. The triangle marks, plus signs and dot marks represent the locations of BSs, AEs and UEs respectively. UEs are non-uniformly distributed in all these models. The scale in both axes is in km.

AE has an individual power constraint. Selective transmission scheme [5] is considered, where a UE is only served by the closest AE within the cell.

A. Network Model

To model uniform and non-uniform distribution of various network elements, homogeneous and inhomogeneous Poisson Point Process (PPP) are used respectively. In this paper UEs are considered to be non-uniformly distributed and modeled as inhomogeneous PPP. For PPP, the number of points m of $\Phi(A)$ in a bounded set $A \subset \mathbb{R}^2$ has a Poisson distribution of mean $\Lambda(A)$ and can be given by [14]

$$\mathbb{P}(\Phi(A) = m) = \frac{(\Lambda(A))^m}{m!} e^{-\Lambda(A)} \tag{1}$$

In case of inhomogeneous PPP, the density measure $\Lambda(A)$ can be expressed as

$$\Lambda(A) = \int \lambda(y) dy \tag{2}$$

where $\lambda(y)$ is the intensity function and y is the location of network nodes along Y-axis. Now, in the area covered by a network, the UE density typically decreases with increasing distance from the most densely populated area. To model this, in our paper we consider the intensity function of UE distribution to be a Gaussian distribution over the region. Hence, the intensity function for UE distribution is expressed as [15]

$$\lambda(y) = \frac{n}{2\pi\sigma^2} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \tag{3}$$

where $n = \Lambda(\mathbb{R})$ is the mean total number of points, σ^2 is the variance of the density over the Euclidean space and μ is the mean location on the plane where density function reaches its peak value.

To model uniformly distributed BSs and AEs, density measure in (1) is taken as $\Lambda(A) = \lambda|A|$, where λ is the density constant of PPP distributed BSs or AEs of DAS over

the terrain. Non-uniform BSs and AEs are modeled using inhomogeneous PPP in similar manner as described in (1) - (3). The variance is considered as equal to the variance of the UE distribution.

We consider four network scenarios with non-uniformly distributed UEs, namely (a) uniformly distributed BS model with no DAS, (b) non-uniformly distributed BS model with no DAS, (c) uniformly distributed BS with uniformly distributed DAS and (d) non-uniformly distributed BS with non-uniformly distributed DAS. In this paper we denote these four network models as *homogeneous BS* model, *inhomogeneous BS* model, *homogeneous DAS* model and *inhomogeneous DAS* model respectively. Snapshots of the aforementioned network layouts are shown in Fig. 1

B. Channel Model

In this paper, we use WINNER+ non-line-of-sight (NLOS) urban macro-cell path-loss model for simulation [13]. According to this model the path-loss P_L in dB is given by the following empirical relationship

$$\begin{aligned} P_L = (44.9 - 6.55 \log_{10} h_{AE}) \log(d) \\ + 5.83 \log_{10} h_{AE} + 14.78 + 34.97 \log_{10} f_c \end{aligned} \tag{4}$$

where d is the distance in metres, h_{AE} is the height of the AEs in metres and f_c is carrier frequency in MHz. For $h_{BS} = h_{AE} = 25m$ and $f_c = 2GHz$, the WINNER+ NLOS path loss formula simplifies to

$$P_L = 138.4 + 35.74 \log_{10}(d) \tag{5}$$

Shadow fading is also considered in this paper and modeled as zero mean log-normal distribution with standard deviation of σ dB. Small scale fading is ignored throughout the paper as OFDM system is considered.

Thermal noise in dB is calculated using Johnson-Nyquist equation. For Δf channel bandwidth in Hz and 300K temperature, the thermal noise power in dBm is given by

$$P_N = -173.8 + 10 \log_{10}(\Delta f) \tag{6}$$

C. Resource Block Allocation

Received SINR at u^{th} UE located in i^{th} BS can be given by

$$\gamma_{i,u} = \frac{\mathcal{P}_{i,u}}{\mathcal{I}_{i,u}^{inter} + \mathcal{I}_{i,u}^{intra} + \mathcal{P}_N} \qquad (7)$$

where $\mathcal{P}_{i,u}$, $\mathcal{I}_{i,u}^{inter}$, $\mathcal{I}_{i,u}^{intra}$ and $\mathcal{P}_N$ are the received power, inter-cell interference, intra-cell interference and the additive white Gaussian noise (AWGN) power respectively. In OFDMA-based LTE networks, resource blocks (RBs) are orthogonal and therefore no intra-cell interference occurs. Considering adaptive modulation and coding (AMC), spectral efficiency (SE) in bps/Hz can be modeled with received SINR $\gamma_{i,u}$ as below [16]

$$\psi_{i,u} = \begin{cases} 0 & if\ \gamma_{i,u} < \gamma_{min} \\ \xi \log_2(1 + \gamma_{i,u}) & if\ \gamma_{min} \le \gamma_{i,u} < \gamma_{max} \\ \psi_{max} & if\ \gamma_{i,u} \ge \gamma_{max} \end{cases} \qquad (8)$$

where $0 \le \xi \le 1$, γ_{min}, γ_{max} and ψ_{max} represent the attenuation factor accounting implementation loss [16], minimum SINR, the SINR at which maximum SE is achieved and maximum SE respectively. Hence, if $R_{i,u}$ is the required data rate in bps, W_{RB} is the bandwidth per RB in Hz (e.g., 180 kHz in LTE), the number of required RBs for the UE for maintaining a particular data rate can be approximated by

$$\beta_{i,u} = \left\lceil \frac{R_{i,u}}{W_{RB}\psi_{i,u}} \right\rceil \qquad (9)$$

where $\lceil x \rceil$ is the nearest integer equal to or larger than x. Now, by fixing the number of RBs per UE, the required SINR for a given data rate can be calculated using (8) - (9).

D. Outage Probability

Let, the required minimum (i.e., threshold) SINR for effective communication is γ_{th}. Then the outage probability of uth UE located in i^{th} cell can be expressed as

$$P_{i,u}^{out} = Pr\{\gamma_{i,u} \le \gamma_{th}\} \qquad (10)$$

where $\gamma_{i,u}$ is the received SINR of uth UE as described in (7). The overall outage probability of the entire network can be evaluated by taking the network-wide average for all the UEs and the cells.

III. RESULTS AND ANALYSIS

A. Simulation Setup

In this paper, we consider a network layout covering a geographical area of 10x10 km^2. Monte Carlo simulation is then used for investigating the network performances. BSs and AEs of DAS are assumed to be equipped with omnidirectional antennas with carrier frequency of 2GHz, channel bandwidth per cell equal to 5MHz (i.e., 25 RBs). BS transmit power or total power of all the AEs in a cell is 20W. The number of RBs in a cell is 25 giving a transmit power per RB equal to 0.8W. We assume that only one RB can be allocated to each UE. AMC code set parameter $\xi = 0.75$, $\gamma_{min} = -6.5dB$, $\gamma_{max} =$

19dB, $\psi_{max} = 4.8bps/Hz$ are chosen in reference to the 3GPP LTE suggestions [16]. Unless otherwise specified, in this paper we consider 512kbps class and the threshold SINR for this class, calculated using (8) - (9), is 11.1 dB, i.e., any UE with received SINR below this value is considered to be in outage.

For homogeneous network model, we consider BS density, $\lambda_{BS} = 1$ per km^2, density of AEs of DAS, $\lambda_{AE} = 15$ per km^2. For inhomogeneous network model, the variance of user distribution as well as non-uniform AE distribution in DAS is equal to 15. Mean total number of points for BSs and AEs are considered to be 188 and 2740 respectively as calculated using (2) to keep the density measure equal to that of homogeneous network model for fair comparison. The standard deviation of shadow fading σ is assumed to be 8 dB, unless otherwise stated.

Network performance is evaluated assuming worst case scenario where all the RBs in every cell are allocated to the UEs, which corresponds to the most severe inter-cell interference scenario. It is also assumed that UEs receive interference from all the cells of the network layouts.

B. Result Analysis

1) Comparative Study of CDF of Received SINR: Fig. 2 presents the cumulative distribution function (CDF) of received SINR giving insight into the distribution of SINR throughout the considered homogeneous and inhomogeneous network models. Both homogeneous and inhomogeneous network models with no DAS show almost identical received SINR distributions. In case of both of the models, received SINR ranges from -58 dB to 77dB. No improvement is observed with the use of inhomogeneous BSs replacing homogeneous BSs. But received SINR is found to improve significantly with the use of DAS. In case of homogeneous DAS model, the received SINR ranges from -52dB to 118dB. On the other hand, received SINR ranges from -44dB to 128dB for inhomogeneous DAS. This implies that, considering non-uniform UE distribution, substantial enhancement of received SINR can be achieved by using inhomogeneous AEs with inhomogeneous BSs compared to the other network models.

2) Downlink Outage Analysis: Fig. 3 illustrates the outage probability of all four network models with varying threshold SINR. As expected, outage probability is found to increase with threshold SINR. It is observed that both homogeneous and inhomogeneous BS models show almost same outage probability which is 72% for the threshold SINR of 11.1 dB. The outage probability improves to 35% for homogeneous DAS model and 32% for inhomogeneous DAS model. It can be stated that, inhomogeneous DAS yields better outage performance than homogeneous DAS. As the AEs in inhomogeneous DAS model are designed according to the non-uniformity of UE distribution, signal strength of cell-edge users improves significantly. This improves overall outage probability in the inhomogeneous DAS model.

In Fig. 4, the impact of AE density of DAS is observed. As before, for all the density measures, the numbers of AEs
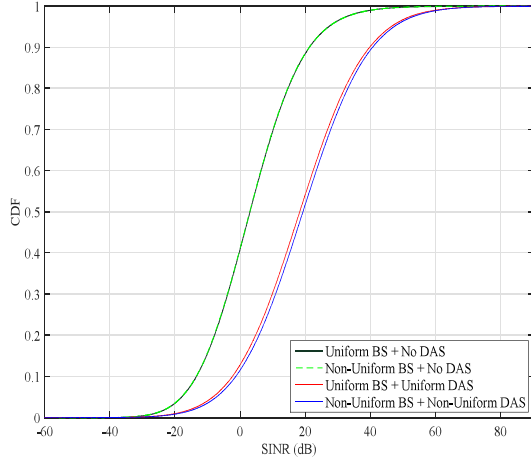
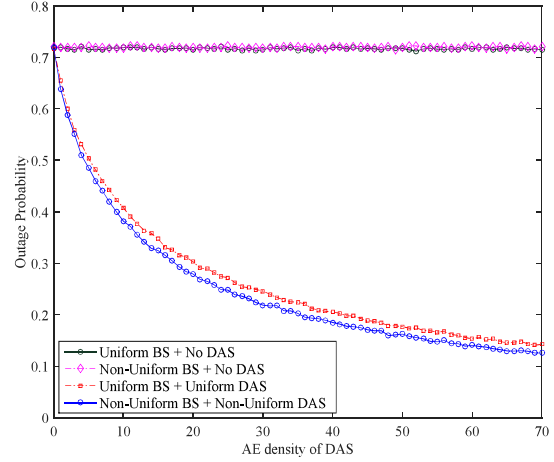Fig. 2: CDF of received SINR for various network layouts.

Fig. 4: Outage probability with varying AE density λ_{AE} of DAS for various network layouts.
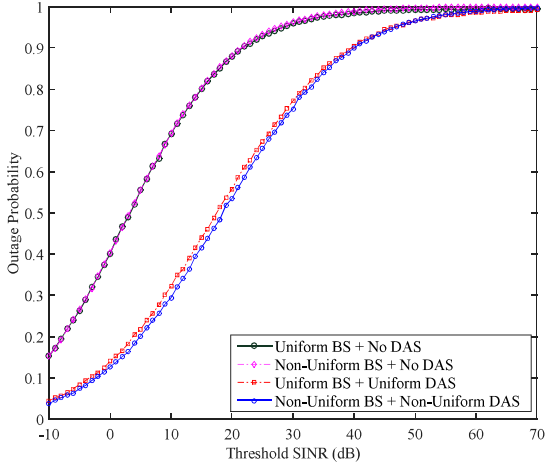

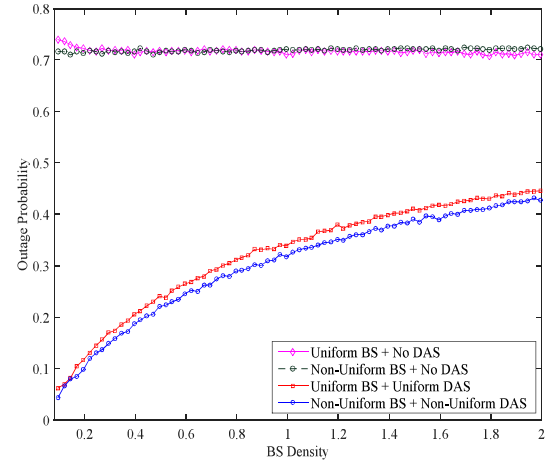Fig. 3: Outage probability with varying threshold SINR for various network models.

Fig. 5: Outage probability with varying BS density λ_{BS} for various network models.

in both homogeneous and inhomogeneous DAS models are kept equal for fair comparison. With the increment of AE density (i.e. $\Lambda_{AE}(A)$ for inhomogeneous DAS and $\lambda_{AE}|A|$ for homogeneous DAS), the outage probability decreases in both DAS models. Increased density of AE, decreases the statistical distance of UEs from service AEs and therefore decreases outage probability. On the other hand, as inhomogeneous DAS follows the distribution of UEs, it significantly decreases the statistical service distance and thereby increases the received SINR compared to the homogeneous DAS model, hence providing improved outage probability in case of inhomogeneous DAS for any particular AE density.
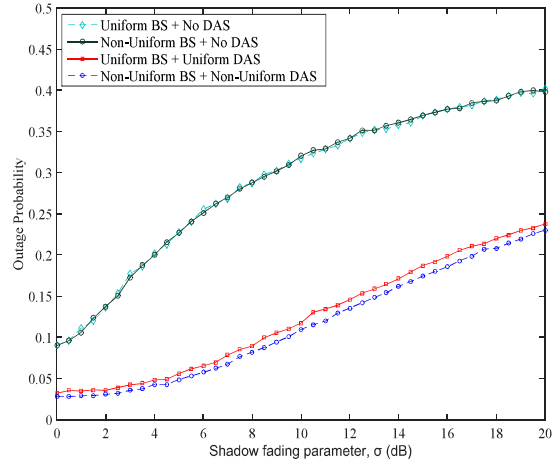
On the other hand, Fig. 5 illustrates the outage probability of all four models with varying BS density. Although for both BS models, outage probability remains almost constant with BS density, λ_{BS}, it increases for DAS models. This can be explained with the definition of SINR. Higher BS density results in an increase in the number of cells over the terrain and a decrease in their respective areas. In the worst case scenario, closely located cells cause more interference and therefore increase outage probability in DAS models. The inhomogeneous DAS model still gives better performance because of its shorter service distance.

Fig. 6(a) and 6(b) depict the outage probability with the varying shadow fading standard deviation, σ for 512kbps and 64kbps service classes respectively. Shadow fading with zero mean lognormal Gaussian distribution has equal probability of increasing or decreasing the received signal and the interfering signal. From the Fig. 2, for BS models 75% of UEs are in
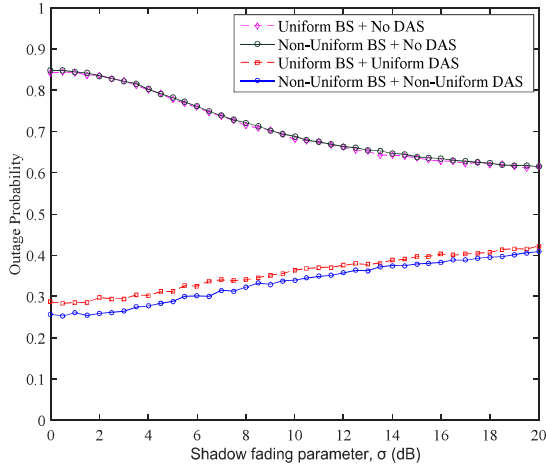
(a) Outage probability for 512kbps class   (b) Outage probability for 64kbps class

Fig. 6: Outage probability of different service classes with shadow fading standard deviation for different network layouts.

outage for the threshold SINR of 11.1 dB. The number of UEs that surpass the threshold SINR due to shadow fading is greater than the number of UEs that fall beneath the threshold SINR. The opposite effect occurs for 64kbps class with threshold SINR of -4.1 dB where only 30% UEs are in outage. On the other hand, both of the DAS models exhibit less variation in outage performance with change in shadow fading parameter. Thus DAS models demonstrate robustness over various terrain profiles for different service classes.

IV. CONCLUSIONS

In this paper, we have proposed and investigated the downlink performance of OFDMA-based LTE cellular networks with DAS considering non-uniformly distributed UEs in space. Various network models with uniform and non-uniformly spatially distributed AEs and BSs have been throughly analyzed. To relate to practical situations, we have modeled the non-uniform UEs using inhomogeneous PPP. Assuming worst case scenario in terms of interference, we have calculated practical lower bounds for the performance metrics of the systems, namely received SINR and outage probability. Simulation results clearly show that inhomogeneous deployment of network elements yields superior performance over traditional homogeneous network distribution on the basis of evaluated outage probability and received SINR.

In future we intend to focus research on analytically modeling this network configuration to simplify analysis and finding cost efficient solutions for distributed antenna system models.

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