Demand Management using Utility based Real Time Pricing for Smart Grid with a New Cost Function

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***Abstract*—Considering the time-varying power consumption of users and cost of generation over a day, demand side management (DSM) has become essential to meet the excessive need of users with the limited source of power. In this paper, we propose a utility based optimal Real-Time Pricing (RTP) mechanism for the future smart grid communication systems such that the electricity price corresponds to the optimum system welfare. Here, we formulate a distributed algorithm which is based on the two-way communication among users, decision maker, and energy provider through the exchange of control messages, and determine the optimal price maintaining the equality between the total demand and the offered generation. We also propose a novel cost function for energy provider exhibiting how it reduces the impact of the change in user number to electricity price, unlike a previously proposed cost function. Simulation results confirm that the proposed algorithm is favorable for both the users and energy provider in terms of electricity price and generation cost respectively. It is also demonstrated that our new cost function makes the RTP algorithm user-adaptive and offers a better welfare to both the users and the energy provider.**

***Index Terms*—Smart Grid, Smart Home, Real Time Pricing (RTP), Demand Side Management (DSM), Dual Decomposition.**

I. Introduction

The smart grid system is regarded as the next generation power grid, where communication technology is integrated into the power grid for its autonomous operations [1]. The Smart home is conceived as an integral part of the total framework where users are allowed to monitor and control their energy consumption in real-time [2]. The system features lots of intelligent networked electronic devices e.g., smart meters, sensor, automatic control devices, etc., which coupled with a demand side management (DSM) strategy enable the grid to use the available energy more efficiently without installing new generation and transmission infrastructure [3], [4]. Present DSM programs aim at solving one or both of the following objectives: reducing consumption and shifting consumption [5]. The former can be achieved among users by encouraging energy-aware consumption patterns and by constructing more energy efficient buildings [6]. However, there is also a need for practical solutions to shift the high-load household appliances to off-peak hours for avoiding over-consumption and under-consumption during peak and off-peak periods respectively [7]. Matching demand and supply of electricity is one of the major challenges for the future smart grid systems.

Taking this into consideration, many DSM techniques have been proposed in the smart grid literature. One of the strategies is Direct Load Control (DLC) [8], where based on an agreement between the utility company and the users, the utility company can control the operations and energy consumption of particular appliances in a household. However, users' privacy is a major concern in employing DLC programs. In [5], a DSM scheme for cooperating customers is proposed and solved in the context of an energy consumption game. The outcomes of the distributed algorithm in [5] are fewer customer charges and a smaller peak power demand. On the other hand, a utility function approach is adopted in [9], where a distributed algorithm is proposed for Real-Time Pricing (RTP). A key feature of the mechanism in [9] is its achievable optimality without revealing customer related parameters, which are assumed to be private. This approach is, by far, the most popular than the other existing DSM mechanisms.

However, none of the approaches consider the impact of a change in user numbers to electricity price. In [9], the system model considers a fixed number of users. If the number of users increases in the system, the algorithm in [9] penalizes all the users by price increment or by making the users consume less amount of power, which is not rational. A more reasonable option is to charge the users with higher unit price only if the demand increases because of their individual increase in demand, not because of the increase in user number. In light of this, we propose a novel utility based optimal RTP mechanism based on the two-way communication infrastructure envisioned in the smart grid, which eliminates the impact of the change in user number.

The contributions of this paper can be summarized as follows.

- We propose a utility based RTP framework for DSM management in future smart grid systems. To do so, users, the decision maker, and the energy provider communicate with each other through two-way message exchanges over a cellular network requiring no human intervention.
- A new cost function for energy provider is proposed and integrated into the proposed framework, which eliminates the impact of the change in user number to real-time price.
- We formulate the RTP as an optimization problem such that the total demand and generation of electricity match. We find each user's demand and the energy provider's total offered generation corresponding to the optimal price leading to the optimal system welfare.
- The proposed framework includes constraints to limit the total energy consumption level of all users to the total electricity generation capacity of the system offered by the energy provider.

The remainder of this paper is organized as follows. In Section II, we propose our RTP based DSM system including the formulation and implementation of our optimization problem. Simulation results and analysis are given in Section III. We conclude the paper in section IV.

II. Proposed Real Time Pricing (RTP) Based Demand Side Management (DSM) System

A. System Model

We consider a smart power system with an ideal cellular network based two-way communication infrastructure, which consists of a single energy provider, several users and a decision maker. In Fig. 1 the whole system model is depicted where the numbers indicate the interaction between users, energy provider, and decision maker serially. Information flow corresponding to the numbers in Fig. 1 are explained as below.

1) Initial price is sent from the decision maker to BS and energy provider.
2) Price update from BS to users (smart homes).
3) Maximum demand update from users to BS.
4) Total demand data from BS to decision maker.
5) Update of offered generation corresponding to the maximum welfare from energy provider to decision maker. Decision maker then updates price.

Then, Step 1 to 5 are repeated until the price is optimal, which is the price at which the total demand and the offered generation reconcile.

B. User's Utility Function

'Utility' is an economic term referring to the total satisfaction received from consuming a good or service. A consumer's utility is hard to measure but it can be determined indirectly with consumer behavior theories which assume that consumers will strive to maximize their utility. The different response of different users to various price scenarios can be modeled analytically by adopting the concept of utility function from microeconomics [10].

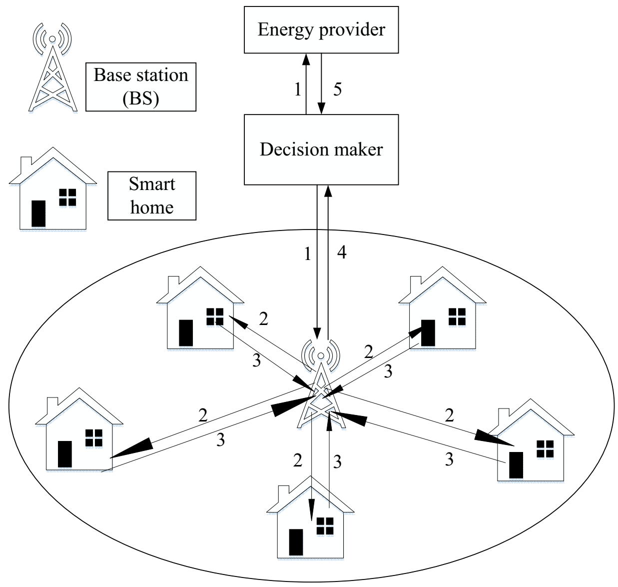

Fig. 1: System model of the proposed RTP based DSM.

In light of this, we assume that the users' utility function fulfills the following properties:

- Every user has a level of consumption different from other users and that changes from a time interval to another.
- A minimum consumption is to be satisfied in all time intervals; while a maximum consumption is the consumption level when all electricity needs of a user are satisfied during a time interval.
- Users are always interested in consuming more until reaching their maximum needs which means the utility function is non-decreasing.
- The increase in the utility function of a user when getting the first units of electricity is much higher than the increase in the same utility function when receiving the $n^{th} (n > 1)$ unit of electricity. In other words, the level of satisfaction gets saturated over time. So, the marginal benefit of users is a non-increasing function. Again, it is convenient to have linear marginal benefit. So the 1^{st} derivative of utility function should be linear and negative.
- Zero consumption means zero utility.

To satisfy the assumptions, the utility function should be quadratic corresponding to linear decreasing marginal benefit, We consider the following utility function [9]

$$U(x,w) = \begin{cases} wx - \frac{\alpha}{2}x^2 & \text{if } 0 \le x \le \frac{w}{\alpha} \\ \frac{w}{\alpha} & \text{if } x \ge \frac{w}{\alpha} \end{cases} \tag{1}$$

where x denotes power consumption level of the user, w is a parameter which may vary among users and also at different times of a day, α denotes a predetermined parameter and Marginal utility $= \frac{\delta u}{\delta x} = -\alpha x$; which is linear and the slope is negative.

If a user consumes x kW electricity at an hour and price of each unit is P, then the welfare of each user can be given by

$$W(x,w) = U(x,w) - Px \tag{2}$$

For each unit price P, each user will try to adjust its power consumption x to maximize his own welfare.

C. Energy Provider's Cost Function

The cost function for electricity produced using thermal generators is represented usually using a quadratic or piecewise linear function. In this paper, we adopt the following quadratic cost function [5], [9], [11]

$$C(L) = aL^2 + bL + c \tag{3}$$

where $C(L)$ denotes the operating cost of generation, L denotes electrical power output (power generation), and a, b and c are the fuel cost coefficients($a > 0$ and $b, c \ge 0$). Energy sources such as solar, wind and hydro are not included in (3) because the fuel that drives its power generation is without a price.

In [9], the coefficients are set to $a = 0.01, b = 0$ and $c = 0$. For a particular price, the energy provider tends to expend a specific amount of cost corresponding to its maximum welfare. Since the coefficients in (3) are considered constant in [9], the energy provider offers a fixed generation of electricity for a fixed cost irrespective of change in user number. However, with the increase of user number, demand gets increased. If we use fixed coefficients, offered generation by the provider will not increase for a fixed electricity price. So, the users will be forced to reduce their demand or they will be penalized with the price increment. But it is not rational to penalize the users for the increase of user numbers. Hence, we propose a new cost function which is user adaptive, that is to say, with the increase of user number, the energy provider increases its offered generation maintaining a fixed price. We use the user number N as a variable such that the cost of generation per user does not change and the offered price remains fixed as long as the total demand is within the maximum generation capacity of the energy provider. Therefore, the users do not get punished for the increment of the user number, rather they are penalized only when they raise their individual demand. However, if the total demand goes beyond the maximum generation capacity, the price has to be increased to keep the demand under control. Our proposed cost function is as follows

$$C(L) = \frac{dL^2}{N} \tag{4}$$

where d is a predetermined parameter that can adjust the optimized price for a fixed set of parameters. When the value of d and N are fixed and same, using (3) or (4) generates the same real-time price.

D. Optimization Problem

Our objective is to maximize the welfare of the users in the grid system and minimize the generation cost of energy providers [12]. The two optimization problems can be merged into one maximization problem that is to maximize the overall welfare. Welfare is in our case defined as the difference between the sum of utility functions of all users and the cost of generation (that depends itself on the number of units produced). But the objective function is subject to several constraints such as ensuring the minimum electricity needs of all users over all time periods. Another constraint would be having the sum of all consumptions of the users less or equal than the maximum generation capacity of the energy provider. Therefore, the optimization problem is

$$max \left[\sum_k \sum_{i=1}^{N} U(x_i^k, w_i^k) - C_k(L_k) \right] \tag{5}$$

where k denotes one time slot. Equation (5) is subject to

$$\sum_{i \epsilon N} (x_i^k) \le L_k \tag{6}$$

and

$$m_i^k \le x_i^k \le M_i^k \tag{7}$$

where m_i^k and M_i^k denote the minimum and the maximum demand of the i^{th} user.

E. Problem Solving: Dual Decomposition Approach

We solve the optimization problem by dual decomposition approach. The dual approach solves the problem in a distributed fashion using a distributed algorithm. A part of the optimization algorithm will run at the side of the users, and the second part of the algorithm will be solved by the energy provider. None of the two parts of the optimization algorithm can be solved independently since the utility parameter for each user is private. Hence a continuous two-way communication system should be established.

The optimization problem is a quadratic convex maximization problem. A way to merge the objective function with the constraint is the method of Lagrange Multipliers [13]. For instance, the Lagrange function for optimization problem maximizing $f(x,y)$ subject to $g(x,y) = 0$ is

$$L(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$

where λ is the Lagrange multiplier. According to this, the Lagrangian for our optimization problem including the constraint is

$$L(x, y, \lambda) = \sum_{i=1}^{N} U(x_i^k, w_i^k) - C_k(L_k) - \lambda^k(\sum_{i=1}^{N} x_i^k - L_k)$$

After re-arranging the equation, we get the following

$$L(x, y, \lambda) = \sum_{i=1}^{N}(U(x_i^k, w_i^k) - \lambda^k x_i^k) + \lambda^k L_k - C_k(L_k) \quad (8)$$

Users will optimize

$$\sum_{i=1}^{N}(U(x_i^k, w_i^k) - \lambda^k x_i^k) \quad (9)$$

Energy providers will optimize

$$\lambda^k L_k - C_k(L_k) \quad (10)$$

where λ^k is the Lagrange multiplier. If the energy provider charges the user at a rate $P = \lambda^{k*}$ and each individual user tries to maximize its own welfare function, it will be guaranteed by strong duality that the total power demand will not exceed the offered power generation of the energy provider.

F. Updating the Price

In our proposed model, price will be updated in an iterative manner until two consecutive prices converge. We use the following equation for updating the Lagrange Multiplier in (8) which is described as the per unit electricity price

$$\lambda_{t+1}^k = \lambda_t^k + \gamma\Big[\sum_i (x_i^{k*}(\lambda_t^k) - L_t^*(\lambda_t^k))\Big] \quad (11)$$

where t is a particular time instance at which price is updated. x_i^{k*} and L_t^* are the optimized value from users and the energy provider at instance t and γ is a pre-determined step size.

Price updating step from user's side is demonstrated in Fig. 2. As shown in the figure, we commence the iteration assuming an arbitrary price. For this price, each user and the energy provider solve their own welfare maximization problem and send their respective results to the decision maker. It means that each user sends its demand (point F) corresponding to the maximum welfare (point A) at that particular price. The energy provider performs the same operation and sends its offered generation. Then based on the difference between the total demand and offered generation, decision maker updates the price using (11). If the difference between the two consecutive prices is smaller than an assumed very small value described as price accuracy, total demand and offered generation match, and we consider the updated price as the optimal one. Otherwise, the iteration goes on (from point B to C and so on) until the difference between two successive

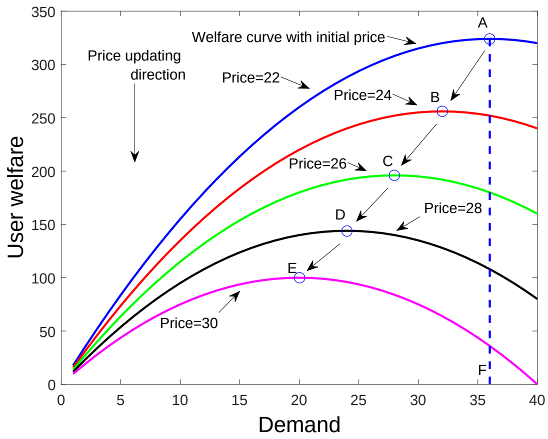

Fig. 2: Principle of demand updating by a user.

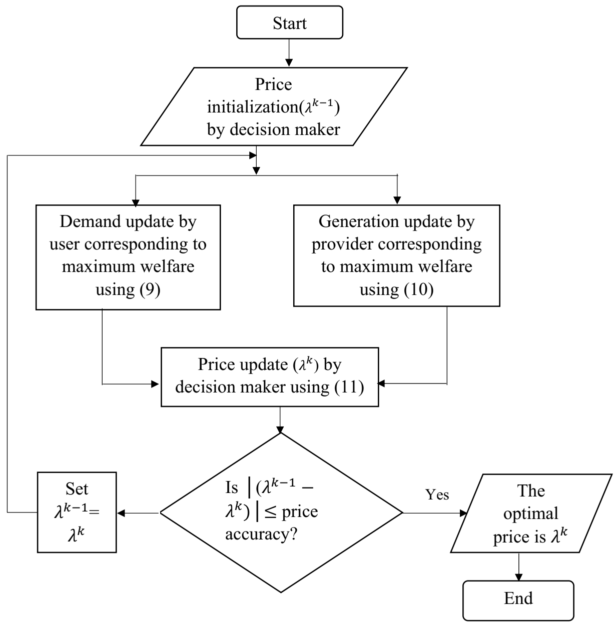

Fig. 3: Illustration of the operation of the proposed algorithm showing the interactions among users, decision maker and energy provider in the system.

prices is smaller than price accuracy. The flowchart of solving the optimization problem is given in Fig. 3.

III. Results and Analysis

In this section, we present our simulation results and evaluate the performance of our proposed distributed algorithm.

A. Simulation Setup

- Minimum and maximum demand of all users are assumed 1 and 320 units respectively.

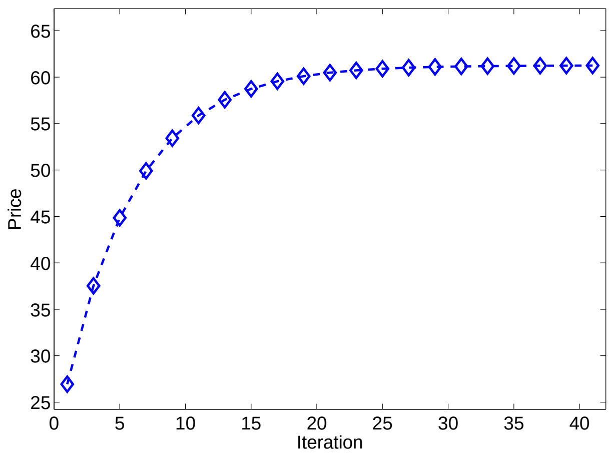

Fig. 4: Update of price with iteration.

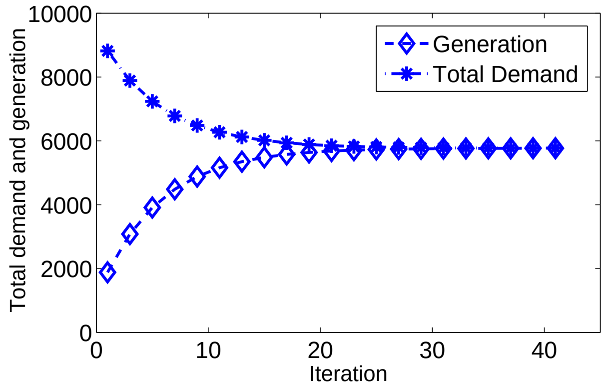

Fig. 5: Total demand and offered generation with iteration.

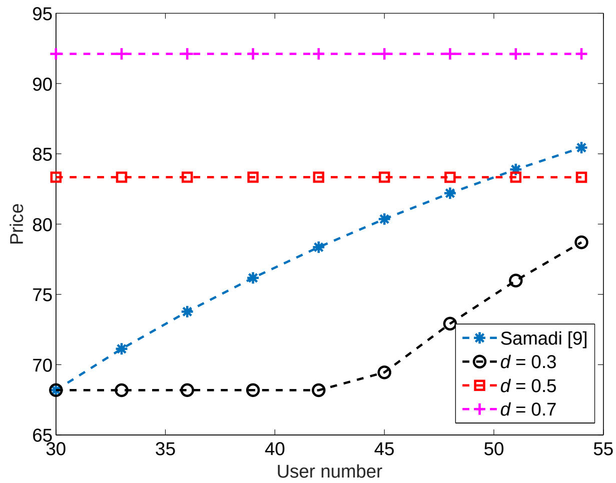

Fig. 6: Optimum price with the number of user.

- The maximum generation capacity of energy provider is assumed 10,000 units. It can supply the maximum demands of 31 users.
- When a user meets its maximum demand, its utility is the highest. From the users' utility function (1), when demand $x \geq \frac{w}{\alpha}$, the utility is the highest. Since we assume $\alpha = 0.5$, if we select $w = x_{max}/2$, we get the maximum demand. Therefore, we select w randomly from $[1, x_{max}/2]$. Random selection of w shuffles the user demand between the minimum and the maximum range.
- Step size is assumed $\gamma = 0.001$.
- Price accuracy is set at 0.002. Iteration continues till the difference between two consecutive prices is less than 0.002.
- We use the *Quadprog* function provided by MATLAB for maximizing welfare at a certain price. *Quadprog* is a minimization tool. So the maximization problem is turned into a minimization one by Max(f) = -Min(-f).
- We assume a low initial price of 20 unit before beginning the iteration.

B. Analysis

Simulation results illustrating the iteration process of our algorithm are from Figs. 4-5. For this purpose, we consider $N = 40$ users in the system. As we start our simulation assuming a low initial price, demand is found much larger than the initial generation as seen in Fig. 5. Hence price goes on increasing at each iteration (Fig. 4) based on RTP algorithm reducing the difference between the demand and the generation of electricity. The simulation runs till demand and production match as shown in Fig. 5. The price corresponding to this situation is the optimal price.

We also verify the performance of our proposed mechanism in Figs. 6-8 illustrating the change of unit price, user welfare and provider welfare with the change of user number.

As seen in Fig. 6, the price goes on increasing with the number of users for the energy cost function used in [9]. Since increased user number means increased demand, the price is raised so that the total demand reduces and matches the offered generation. However, the price should be kept constant with the change of user number so that the users are not penalized unjustifiably. Our proposed energy cost function is capable to keep the unit price stable in this case (Figs. 6-8). We also illustrate how the value of d can play a role to set the optimized price at different values under different circumstances. The higher the value of d, the more the optimized price will be because higher value of d will cause the provider to generate less electricity. Moreover, we observe an increment of price while using $d = 0.3$ when the curve goes beyond the user number $N = 40$. That's because lower value of d causes the optimized price to be lower which motivates the user to consume more and the total demand crosses the maximum generation capacity of the energy provider very quickly with the increase of user number. Consequently, the decision maker has to increase

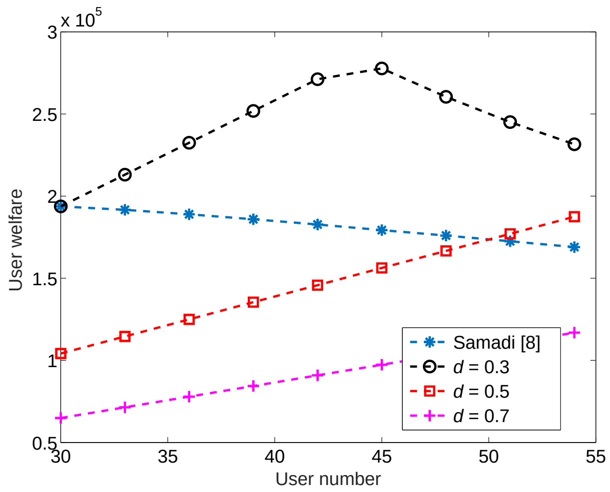

Fig. 7: User welfare with the number of user

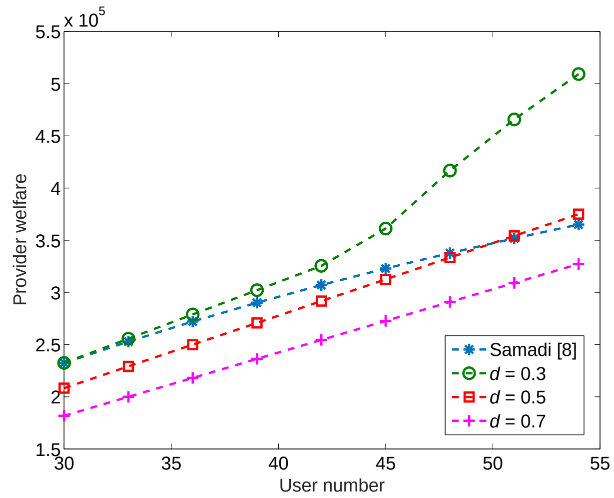

Fig. 8: Provider welfare with the number of user.

the price for limiting the demand.

Furthermore, the user welfare curve while using (3) decreases as the user number increases (Fig. 7) because of the price increment with the user number. On the contrary, the per user welfare while using our proposed cost function (4) remains the same because of the fixed unit price. The overall user welfare curve continues increasing with the user number unless the total demand exceeds the maximum generation capacity since the use of (4) ensures the satisfaction of more users and the price remains fixed. Again, the provider welfare increases in both cases (Fig. 8) since the unit price increases while using (3) and more users can consume electricity while using (4) generating more profit overall for the energy provider. The provider welfare curve becomes more steep when the total demand crosses the maximum generation capacity of the provider. Hence it can be claimed that our proposed cost function is a generalized one; it is more addaptive to different situations and offers better welfare to both the users and the energy provider.

IV. Conclusions

In this paper, we have proposed a utility based optimal RTP mechanism for DSM. The proposed mechanism relies on the two-way communication infrastructure envisioned in the future smart grid. The optimal price has been calculated matching the total demand with offered generation. We have also proposed a novel user-adaptive cost function which can neutralize the impact of a change in user number to the real-time price and offers better welfare for both the users and energy provider. The performance of the proposed mechanism has also been compared with a previously proposed popular work demonstrating the flexibility and superiority of our one. Our future works will consider a scenario with multiple energy providers. The impact of the wireless channel as a communication medium on the performance of the optimal price will also be explored.

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